Fairness in Machine Learning as a Causal Question

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EECS 598-009, Causality and Machine Learning, Fall 2023

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Disclaimer

- Fairness is an inherently normative topic
- Talking about fairness means covering some sensitive topics
- We're all from different backgrounds and probably won't agree on everything, and that's ok

Goals & ground rules

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- **Anti-goal:** Tell you the "right way" to think about fairness.
- **Norms for discussion:** Assume good intent from others, and avoid making broad generalizations.

Today's focus

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Remember the causal inference pipeline...

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- Define the estimand of interest

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- Demonstrate identifiability

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We'll be investigating how we can formulate problems of fairness in machine learning/decision-making as causal questions.

What is fairness in machine learning?

Fairness: A Motivating Example

[1] Angwin et al. (2014), "Machine bias," https://www.propublica.org/article/machine-bias-risk-assessments-in-criminal-sentencing

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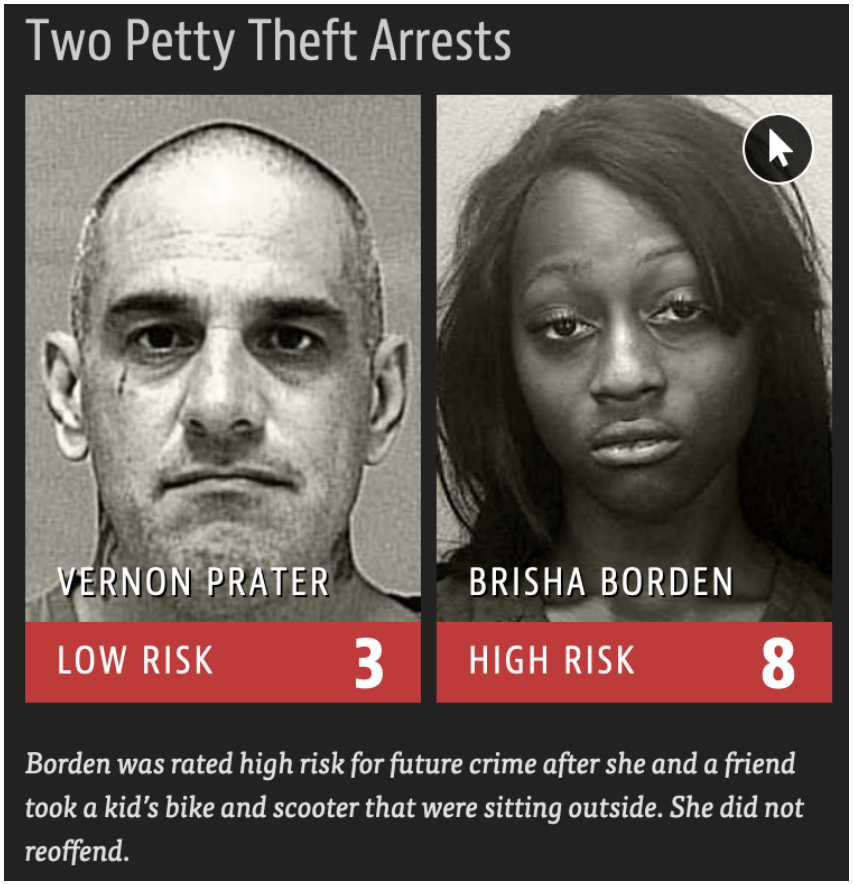

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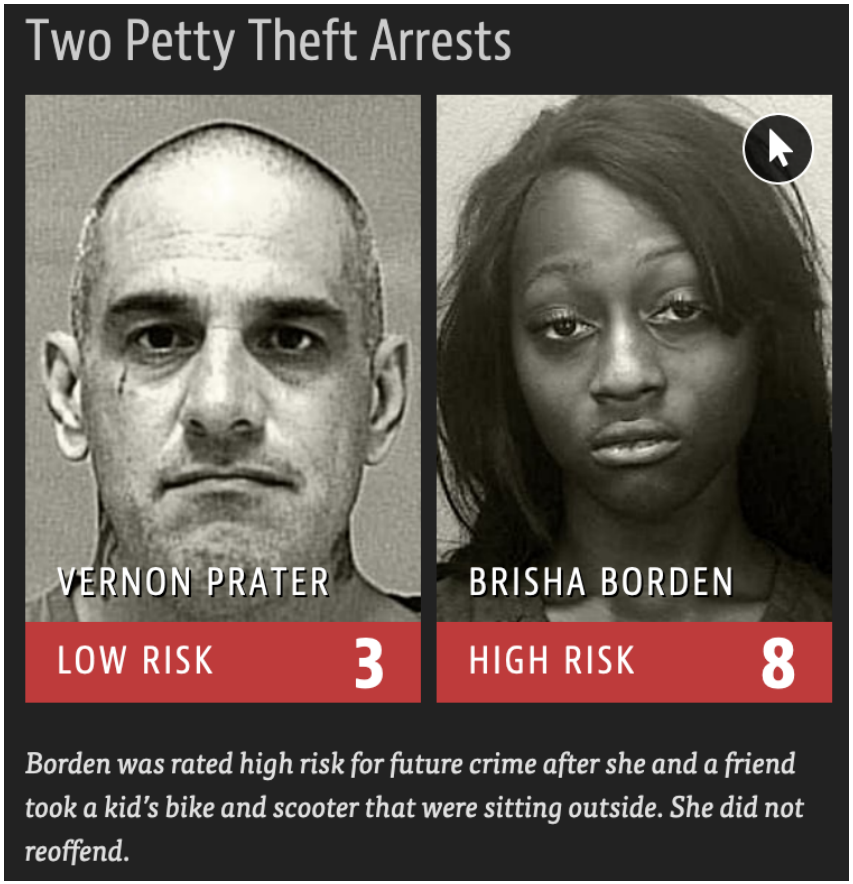

An analysis of the COMPAS redicivism risk prediction algorithm highlighted racial bias in the algorithm's outputted risk scores.[1]

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COMPAS: reconstructed results for Broward County, FL data

[2] Since the algorithm outputs a number 1-10, as well as a bracket "Low," "Medium," and "High," the authors of this analysis treat "Low" as the negative label (did not recidivate) and "Medium/High" as positive. Data reproduced from Larson et al. (2016), "How We Analyzed the COMPAS Recidivism Algorithm," https://www.propublica.org/article/how-we-analyzed-the-compas-recidivism-algorithm.

COMPAS: reconstructed results for Broward County, FL data

We show model[2] performance across groups:

- $\hat{Y} = 0$: predicted to not reoffend
- $\hat{Y} = 1$: predicted to reoffend

Ground truth	White defendants		Black defendants	
	$\hat{Y} = 0$	$\hat{Y} = 1$	$\hat{Y} = 0$	$\hat{Y} = 1$
Did not reoffend	990	805	1139	349
Recidivated	532	1369	461	505

Table: Confusion matrix by race of the COMPAS algorithm.

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ProPublica's argument

ProPublica argued that the model is discriminatory/unfair, because it makes disproportionate errors among Black defendants:

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44.85% vs. 27.99% is a pretty (subjectively) large gap![3]

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OK, 62.97% and 59.13% are relatively close...

OK, now what?

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The trouble with fairness

Clearly, defining "fairness" is subjective. We need some way to formalize our assumptions about what's "fair."

Observational fairness: notation

There are three main categories of **observational fairness** definitions, which we will encode as [conditional] independence relationships between the following variables:

- A: sensitive attribute
- Y: any outcome of interest
- $\hat{Y}$: any prediction of the outcome of interest. Commonly assumed to be some function of a set of covariates X (*i.e.*, a model).

Sensitive attributes

Definition: Sensitive attribute

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Remark

There is no widely-accepted, mathematically-rigorous definition of a sensitive attribute. Its definition originates in anti-discrimination law (in a U.S. context, where it is called a *protected class*[4]), but is generally hand-waved.

[4] See the Civil Rights Act of 1964.

Observational definitions of fairness

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A fairness criterion is **observational** if it can be written in the form $f(P(A; Y; \hat{Y}; X))$ for some functional f.

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Remark

Intuitively, we can express **observational definitions of fairness** in terms of joint/conditional probability statements.

The three categories of observational fairness criteria

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Most observational fairness definitions can be encoded as the following (conditional) independence conditions:

- Independence: $\hat{Y} \perp\!\!\!\perp A$
- Separation: $\hat{Y} \perp\!\!\!\perp A \mid Y$
- Sufficiency: $Y \perp\!\!\!\perp A \mid \hat{Y}$

Independence

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Intuition

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Other names in the literature

- demographic parity, statistical parity, group fairness, disparate impact

How people measure independence

One common empirical measurement of fairness

Follows from the statistical definition of independence; for some pre-specified threshold > 0, we have that

$$\forall (a; a^{0}; \hat{y}): \quad P(\hat{Y} = \hat{y} \mid A = a) \quad P(\hat{Y} = \hat{y} \mid A = a^{0}) \quad :$$

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- Assumes we fully observe all A and $\hat{Y}$.
- Empirical measurements of other fairness criteria proceed similarly for other definitions (adding the assumption that Y is fully observed).
- There are other ways to measure fairness as well (less common in my observation), *e.g.*, MMD, f-divergences, mutual information.

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Other names in the literature

- Error rate parity/equality of error rates, false positive/negative error rate balance, equalized odds. See Verma (2018) for more.[a]

[a] Verma, S., & Rubin, J. (2018). Fairness de nitions explained.

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Other names in the literature

- Statistical calibration, group calibration, predictive parity.

The limits of observational definitions

The 1973 Berkeley admissions case study

Department	Men		Women	
	Applied	Admitted (%)	Applied	Admitted (%)
Total	2651	44	1835	30

[5] Reproduced from Barocas, Hardt, and Narayanan (2019).

The 1973 Berkeley admissions case study

Question

Given the information we have, which definition of fairness could we apply to this example?

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The 1973 Berkeley admissions case study

Question

What happens when we apply our fairness definition at the department level?

Department	Men		Women	
	Applied	Admitted (%)	Applied	Admitted (%)
Total	2651	44	1835	30
A	825	62	108	82
B	520	60	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7

Table: UC Berkeley admissions data from 1973[5]

[5] Reproduced from Barocas, Hardt, and Narayanan (2019).

Observational definitions of fairness are not explanations

- When we tried to apply a naive fairness definition to evaluate the fairness of UC Berkeley admissions decisions from 1973, we ran into *Simpson's paradox*.

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- When we tried to apply a naive fairness definition to evaluate the fairness of UC Berkeley admissions decisions from 1973, we ran into *Simpson's paradox*.
- There are a bunch of potential explanations for why this difference occurs, but it is impossible to tell from the table if these are true.
- Observational definitions of fairness can tell us whether a disparity exists, but are *not* explanations.

DAGs to the rescue? Graphical discrimination analysis

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Why DAGS?

The punchline

Thus, **our prior beliefs in \how the world works/should work"** can help us **choose a fairness de nition**—and in turn figure out what constraints we can impose on estimation/modeling.

Turn to your neighbor and discuss a potential DAG for the Berkeley admissions case study. Use (at least) these variables:

A = gender as reported on the application form

X = department choice

Y = admission decision

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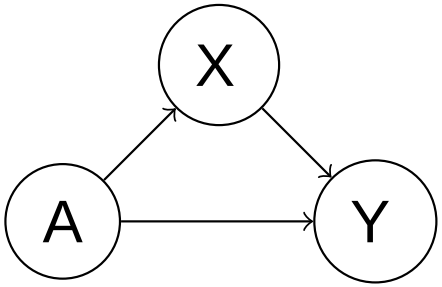

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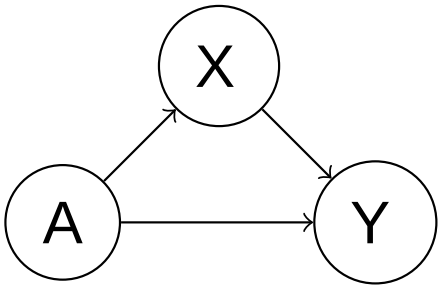

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In causal language, what is one way we could argue that there is/isn't any discrimination?Hint: Think about hypothetical values of causal e ects.

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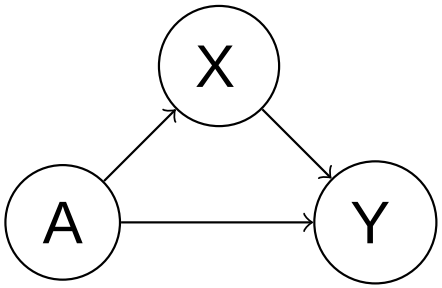

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The causal e ect ofA on Y is zero.

Counterfactual de nitions of fairness

Formally, we might come up with counterfactual versions of observational fairness de nitions by replacing Y with $Y(a)$ in our existing observational de nitions of fairness, e.g.,

$$Y(a) \perp A \tag{1}$$

for counterfactual independence (if the applicant's gender had been di erent from what was observed, their admission status should not change).

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Takeaway

Counterfactual fairness asserts that "if a sensitive attribute had been di erent, there would be no e ect on the outcome. (potentially conditional on other information)"

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Conducting graphical discrimination analysis

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- Using the DAG, we turn identifying fairness or discrimination into identifying a causal effect. We know how to do that!

(one hypothetical fairness measurement in our setting)

$$\frac{1}{N}\sum_{i=1}^{N} P(Y \mid A = \text{male}) \quad P(Y \mid A = \text{female})$$

Interpreting graphical discrimination analysis

Under our causal assumptions, \fairness" is de ned in terms of a null causal e ect.

Counterfactual interpretation: \Intervening" to change a sensitive attribute should not a ect the outcome.

So, we identi ed a causal e ect. Can we go home?

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Under consistency, no unmeasured confounders, and positivity; the causal effect of gender on acceptance to graduate school at Berkeley in 1973 is identifiable.

But this doesn't really explain why/how discrimination arises...

And, doesn't help us with our original issue—even with causal assumptions, it's not clear how we can explain discrimination (yet)!

Structural discrimination: University of Adversaria

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The University of Adversaria systematically reduces funding to programs that attract more female applicants.

This arti cially reduces acceptance rates in such departments.

Structural discrimination, continued

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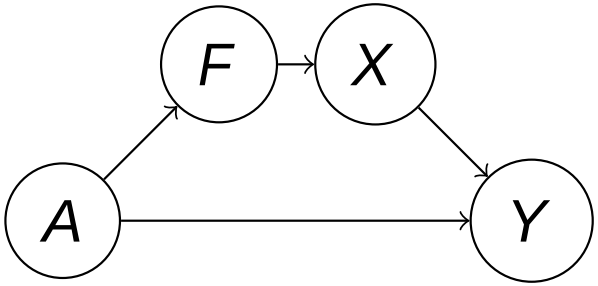

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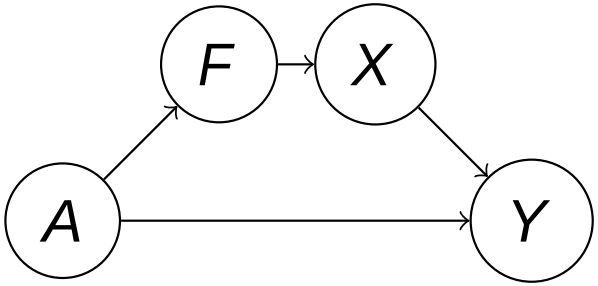

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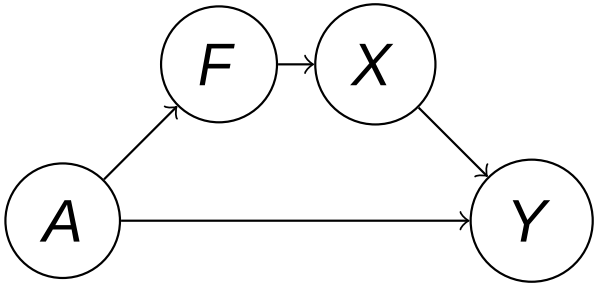

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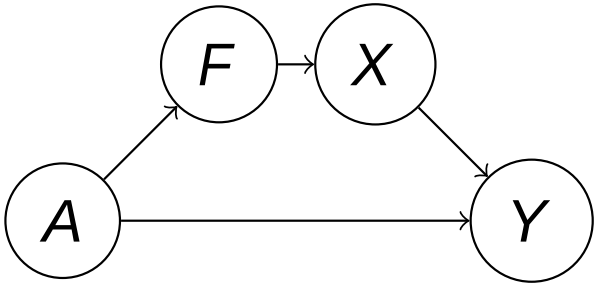

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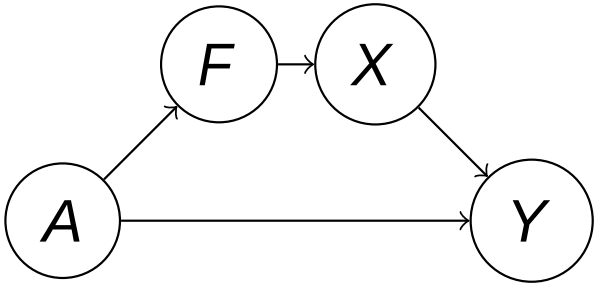

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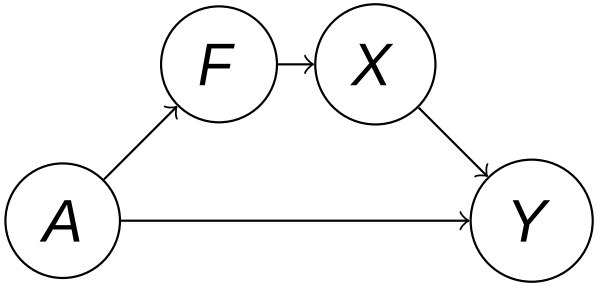

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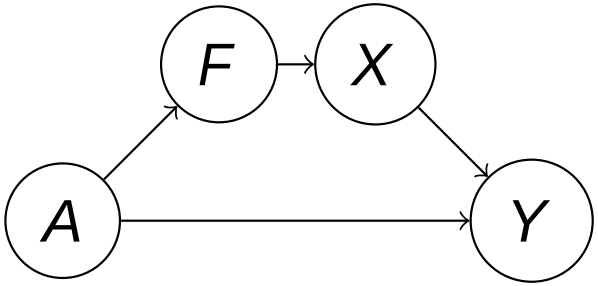

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- $A \; ! \; F \; ! \; X \; ! \; Y$: indirect gender discrimination due to structural factors (*structural discrimination*)

Question

Why might separating out these two sources of discrimination be useful? *I.e.*, isn't all discrimination bad?

Conflating sources of discrimination

Our naive estimate of the effect of A and Y can conflate two sources of discrimination:

- $A \; ! \; Y$: systematic, direct gender discrimination (*taste-based discrimination*)
- $A \; ! \; F \; ! \; X \; ! \; Y$: indirect gender discrimination due to structural factors (*structural discrimination*)

Question

Why might separating out these two sources of discrimination be useful? *I.e.*, isn't all discrimination bad?

Yes, but if we want to design policies that target the underlying *causes* of discrimination, separating these out could be useful.

Discrimination analysis with direct & indirect effects

The natural direct effect

If we care about discrimination along $A \not\perp Y$, we might want to measure, across *fixed* levels of X (or F), the effect of A on Y. Perhaps we can fix the levels to $X(a)$, and aggregate.

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For an arbitrary mediator M (*i.e.*, $M \;2\; fX; Fg$), this is the **natural direct e ect**:[7]

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Natural Direct Effect (NDE)

$$\begin{aligned} & \mathrm{E}[Y(a; M(a^0))] \quad \mathrm{E}[Y(a^0; M(a^0))] \\ = & \sum_m [\mathrm{E}[Y \;j\; m; a] \quad \mathrm{E}[Y \;j\; m; a^0]] P(m \;j\; a) \end{aligned}$$

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NDE Intuition

Let a; a^0 correspond to \male, female," respectively (without loss of generality). Set M := X (we are analyzing department choice as a mediator). The NDE is the di erence between two terms:

[8] Here, \if" is shorthand for a counterfactual; i.e., what would have happened if we intervened such that some variable takes on a speci c value.

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$E[Y(a, X(a))]$: Acceptance status (Y) if[8] male and department choice (X) is what it would be if individual had been male

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$E[Y(a', X(a))]$: Acceptance status if female and department choice is what it would be for a male applicant

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>
> $E[Y(a', X(a))]$: Acceptance status if female and department choice is what it would be for a male applicant

Intuition: The NDE "turns off" the effect of the mediator on the outcome by fixing it given an intervention, such that we only capture the effect of gender directly on admission.

[8] Here, "if" is shorthand for a counterfactual; i.e., what would have happened if we intervened such that some variable takes on a specific value.

The natural indirect e ect

Whew! That takes care of the path $A \to Y$. What about the path $A \to F \to X \to Y$?

For this one, we can turn to the natural indirect e ect. For an arbitrary mediator $M \in \{F, X\}$:

Natural Indirect E ect (NIE)

$$\begin{aligned} & E[Y(a, M(a))] - E[Y(a, M(a'))] \\ = & \sum_m E[Y \mid m, a][P(m \mid a') - P(m \mid a)] \end{aligned}$$

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Setting $M := X$ again, let's break down each term of the NIE:

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Setting $M := X$ again, let's break down each term of the NIE:

$E[Y(a, X(a))]$: Acceptance status (Y) if male and department choice X is what it would be if individual had been male (same term as NDE)

$E[Y(a, X(a'))]$: Acceptance status (Y) if male and department choice X is what it would be if individual had been female

What's different from before?

NIE Intuition

Setting $M := X$ again, let's break down each term of the NIE:

- $E[Y(a, X(a))]$: Acceptance status (Y) if male and department choice X is what it would be if individual had been male (same term as NDE)
- $E[Y(a, X(a'))]$: Acceptance status (Y) if male and department choice X is what it would be if individual had been female

What's different from before? For the NDE, the first component of the $Y(\ ,\ )$ counterfactual is different; for the NIE, the $X(a)$ (2nd) component differs.

Intuition: The NIE "turns off" the effect of the treatment directly on the outcome, only allowing it to affect $Y(\ ,\ )$ via changes to the mediator (i.e., to $X(\ )$).

The Mediation Formula (Identi ability of the NDE/NIE)

C: confounder(s) (cfd.),M: mediator (med.)[9]

Assumptions

1. $8a$: $Y(a; M(a)) = Y(a)$ (composition)
2. $8(a; m)$: $A \perp\!\!\!\perp Y(a; m) \mid C$ (no treatment-outcome cfd.)
3. $8(a; m)$: $M \perp\!\!\!\perp Y(a; m) \mid (C; A)$ (no med.-outcome cfd.)
4. $8a$: $A \perp\!\!\!\perp M(a) \mid C$ (no treatment-med. cfd.)
5. $8(a; a'; m)$: $Y(a; m) \perp\!\!\!\perp M(a') \mid C$ (no \cross-world" confounding)

[9] Further reading: Ding (2023), A First Course in Causal Inference, Ch. 27. https://arxiv.org/pdf/2305.18793.pdf

Theorem: Identi ability of the NDE

Theorem

Under the previous assumptions (2-5), we have that

$$E[Y(a, M(a'))] = \sum_m E[Y \mid A = a, M = m] P(M = m \mid A = a').$$

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$$E[Y(a; M(a^0))] = \sum_m E[Y \mid A = a; M = m] P(M = m \mid A = a^0).$$

$$E[Y(a; M(a^0))] = \sum_m E[Y(a; M(a^0)) \mid M(a^0) = m] P(M(a^0) = m)$$

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$$
\begin{aligned}
E[Y(a; M(a'))] &= \sum_{m} E[Y(a; M(a')) \mid M(a') = m] P(M(a') = m) \\
&= \sum_{m} E[Y(a; m) \mid M(a') = m] \underbrace{P(M = m \mid A = a')}_{A? \; M(a) \perp C + \text{consistency}}
\end{aligned}
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&= \sum_m \underbrace{E[Y(a; m)]}_{Y(a; M(a)) \perp M(a^0) \mid C} P(M = m \mid A = a^0)
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&= \sum_m \underbrace{E[Y \mid A = a; M = m]}_{A \perp Y(a;m) \mid C \wedge M \perp Y(a;m) \mid (C;A)} P(M = m \mid A = a^0):
\end{aligned}
$$

□

How focusing on the ATE con ates NDE and NIE

Recall that the motivation for the NDE and NIE was that we were con ating two sources of discrimination:

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- $A \to Y$: systematic, direct gender discrimination (as opposed to caste-based discrimination)

How focusing on the ATE con ates NDE and NIE

Recall that the motivation for the NDE and NIE was that we were con ating two sources of discrimination:

- $A \to Y$: systematic, direct gender discrimination (a.k.a. taste-based discrimination)
- $A \to F \to X \to Y$: indirect gender discrimination due to structural factors (structural discrimination)

It turns out, we can write that the ATE = NDE + NIE:

$$\begin{aligned}\text{ATE} &= E[Y(a)] - E[Y(a')] = E[Y(a, X(a))] - E[Y(a', X(a'))] \\ &= E[Y(a, M(a))] - E[Y(a, M(a'))] + E[Y(a, M(a'))] \\ &\quad - E[Y(a', M(a'))] = \text{NIE} + \text{NDE}\end{aligned}$$

Beyond NIE (informal)

Recall our DAG for this problem..

[10] The identi ability conditions here get somewhat involved; for more details, consult Nabi and Shipster (2017), \Fair Inference on Outcomes" and Pearl (2005), \Direct and Indirect E ects," Section 3.7.

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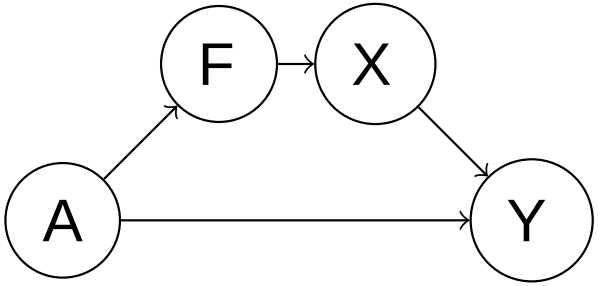

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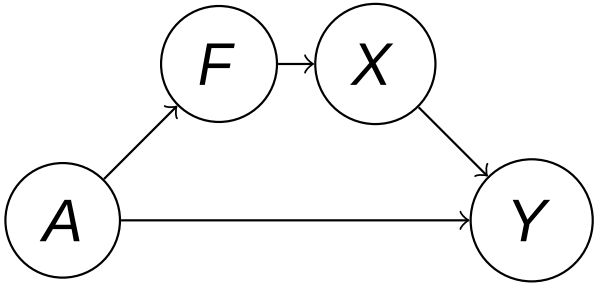

- There are only two paths from $A \; ! \; Y$: $A \; ! \; Y$ itself and $A \; ! \; F \; ! \; X \; ! \; Y$

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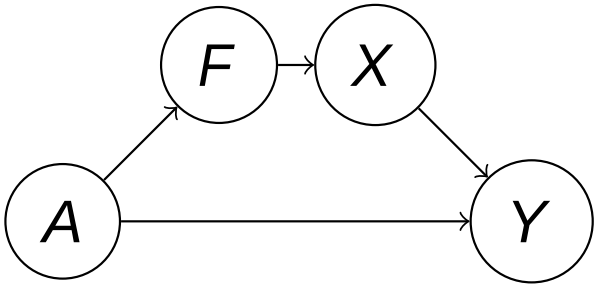

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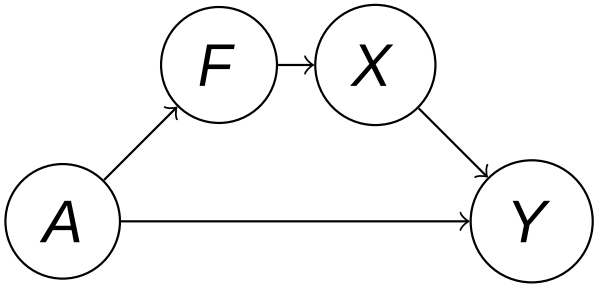

- There are only two paths from A ! Y: A ! Y itself and A ! F ! X ! Y
- If we have more than two paths, we can generalize the NIE to **path-speci c e ects**
- This is done by "turning off" causal effects along all paths—except the one we care about.[10]

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(Bonus) Pitfalls of using sensitive attributes in causal inference

Recall: in causal fairness analysis, what do we usually define as the "**treatment**?"

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A **sensitive attribute!**

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A **sensitive attribute!**

But a treatment is an *intervention*, which raises questions:

- How can we intervene on someone's demographics?
- Is causal inference even well-defined when treatment is defined as a (presumably immutable) sensitive attribute?
- Is this *purely* a philosophical problem, or can it have real implications on causal effect estimation?

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The only thing di erent about these universes is the intervention! This sounds good for something like a clinical trial with an RCT design...

The paradox of sensitive attributes as interventions

Suppose that we have a study to determine the effect of race (White vs. Black) on hiring decisions. We use submitted resumes to collect data.[11]

[11] Loosely based on Bertrand and Mullainathan (2004), \Are Emily and Greg more employable than Lakisha and Jamal?"

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In this case, what do you think "all else being equal" means?

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"All else being equal"

In this case, what do you think "all else being equal" means?

Potential positivity violation—not clear if such an individual exists, nor is *intervening* on race well-defined!

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A potential resolution through social constructivism

[12] Further reading with respect to race: Omi and Winant (1985), Racial Formation in the United States, Ch. 4

[13] Here, I slightly disagree with Kasirzadeh and Smart (2021); see their paper for a counterpoint.

A potential resolution through social constructivism

Main idea: Social categories such as *race* do not have inherent physical grounding, but rather physical/real-world objects are "given" extraneous meaning via societal norms, policies, or laws.[12]

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\We get it Trenton, you're a humanities kid; what does this mean for causal inference?" When we say we want to measure the causal effect of *race*, *gender*, or some other social category on an outcome—*race/gender* are simply *shorthand*/abbreviations for some *aspect* of race/gender/etc.[13]

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Revisiting racial bias in resume screening

We can resolve this issue for the resume screening example as follows:

- "race" *!* "racial perception of name by evaluator"

Takeaways

Be precise!

- Clearly state what *effect* we're trying to measure when we treat a sensitive attribute as a variable.
- This means clearly defining what aspect of a sensitive attribute that you care about (*e.g.*, a decision-maker's *perception* of race, someone's *self-reported* gender, biological sex)

Conclusion: What we learned today

Closing share-out

Turn to a neighbor and discuss what you learned today!

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Closing share-out

Turn to a neighbor and discuss what you learned today!

My takeaways:

- We motivated ways to define fairness from a non-causal and causal perspective
- We discussed how causal fairness is a matter of testing for a null causal effect (if a person had different characteristics, the outcome shouldn't change)
- We highlight different causal effects (vanilla ATE, NDE, and NIE) to estimate when thinking about causal fairness